



# Frequency maps of LHC models

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## Motivation

- Main Issue in Hadron Machines —> Long Term Stability of the Beam  
( $> 10^7$  Turns at Injection for LHC)
- Principal Nonlinearities:
  - a. Dipole Multipole Errors (Injection)
  - b. Low  $\beta$  Quadrupole Errors + Beam-Beam (Collision)
- Tracking Codes (MAD, SIXTRACK) —> Dynamic Aperture ( $10^5$  turns)  
????? Structure of Phase Space ?????
- Normal forms
- Frequency Map Analysis



## Contents

- The Frequency Map Analysis Method
- Application to the LHC
  - a. Injection Optics Version 5
  - b. Injection Optics Version 6
  - c. Beam-Beam
- Conclusion - Perspectives

## Frequency Map Analysis

- Quasi-periodic approximation (NAFF, SUSSIX,...)

$$f'_j(t) = \sum_{k=1}^N a_{j,k} e^{i\omega_{jk} t} \text{ with } f'_j(t), a_{j,k} \in \mathbb{C} \text{ and } j = 1, \dots, n,$$

of a complex function  $f_j(t) = q_j(t) + i p_j(t)$ , for  $t = \tau$

- Determination of the frequency vector  $(\nu_1, \nu_2, \dots, \nu_n)$
- Theorem for precision  $\rightarrow \frac{1}{\tau^{2r+2}}$  (Laskar 1996)

For Hanning Filter where  $r = 2$  the precision  $\rightarrow \frac{1}{T^4}$



Construction of the frequency map  $\mathcal{F}_\tau$ :

$$\begin{aligned} \mathcal{F}_\tau : \quad & \mathbb{R}^n & \longrightarrow & \mathbb{R}^n \\ & \boldsymbol{p}|_{\boldsymbol{q}=\boldsymbol{q}_0} & \longrightarrow & \boldsymbol{\nu} . \end{aligned}$$

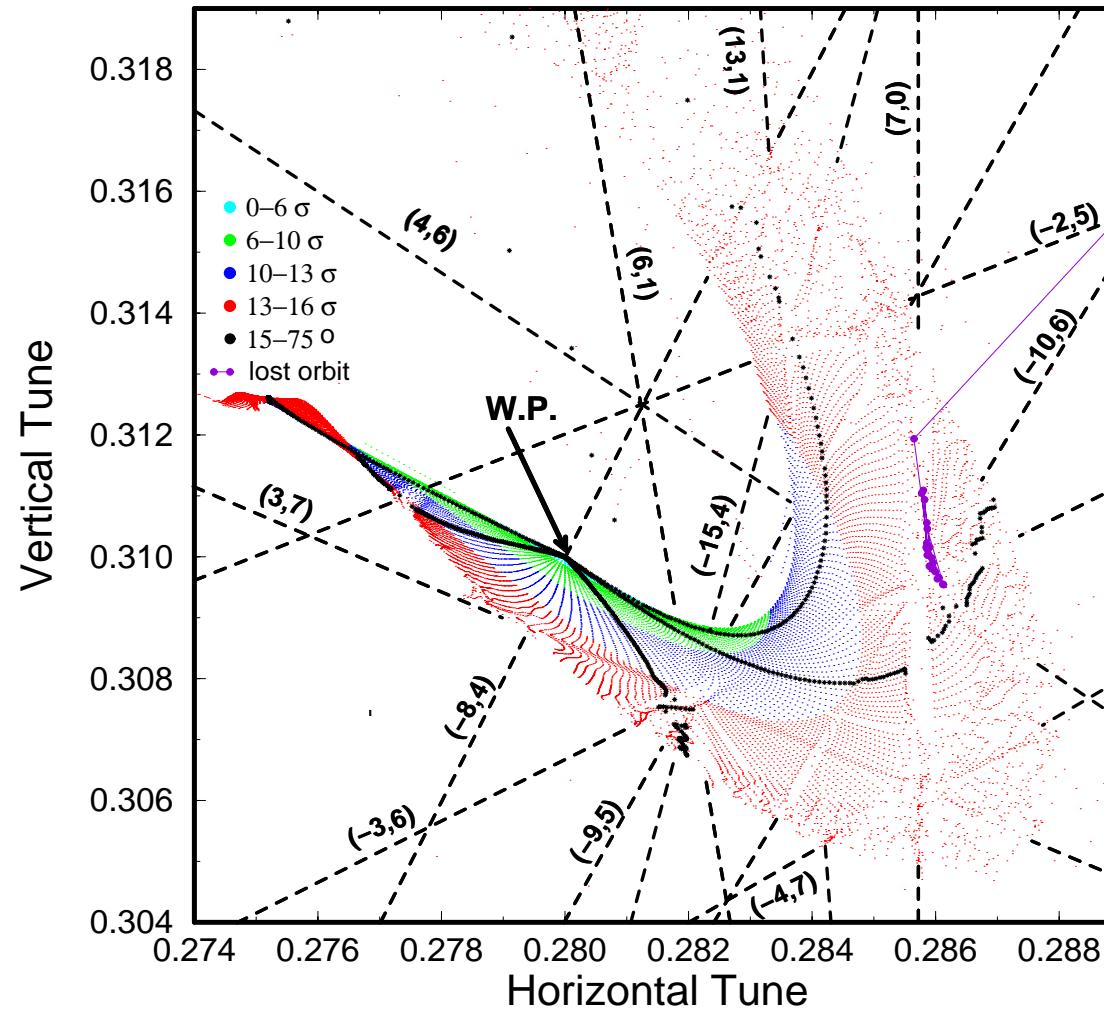


## Frequency Maps for the LHC

- Short-term tracking data ( $\tau = 10^3$  turns) issued by SIXTRACK
- Choose arbitrary section of the phase space  $\longrightarrow p_x, p_y = 0$ , steps in  $I_{x0}$ ,  $I_{y0}$ , at different  $I_{x0}/I_{y0}$

$$\begin{aligned}\mathcal{F}_\tau : \quad & \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ & (I_x, I_y)|_{p_x, p_y=0}, & \longrightarrow & (\nu_x, \nu_y)\end{aligned}$$

## LHC V5 Target Table





## Diffusion

- Diffusion Coefficient

$$D_c = \lim_{n \rightarrow \infty} \frac{\langle (p_n - p_0)^2 \rangle_R}{n}$$

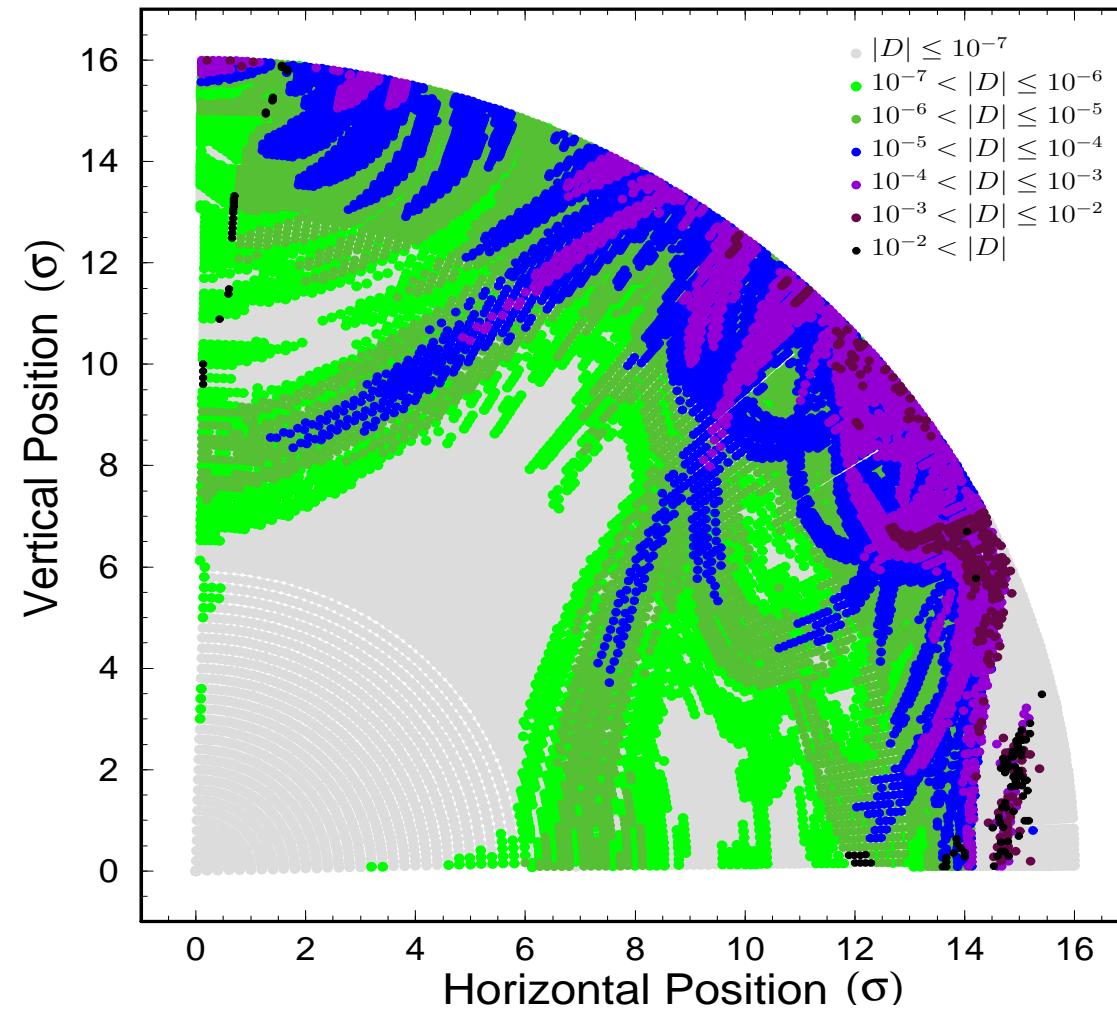
- Frequency Diffusion Vector

$$\mathbf{D}|_{t=\tau} = \boldsymbol{\nu}|_{t \in (0, \tau/2]} - \boldsymbol{\nu}|_{t \in (\tau/2, \tau]}$$

- Diffusion Quality Factor

$$D_{QF} = \left\langle \frac{|\mathbf{D}|}{(I_{x0}^2 + I_{y0}^2)^{1/2}} \right\rangle_R .$$

## Diffusion Map





## Effect of Skew Octupole Error

- Skew octupole error  $a_4^{9712}$  are 5 times bigger than  $a_4^{tar}$  in LHC Optics

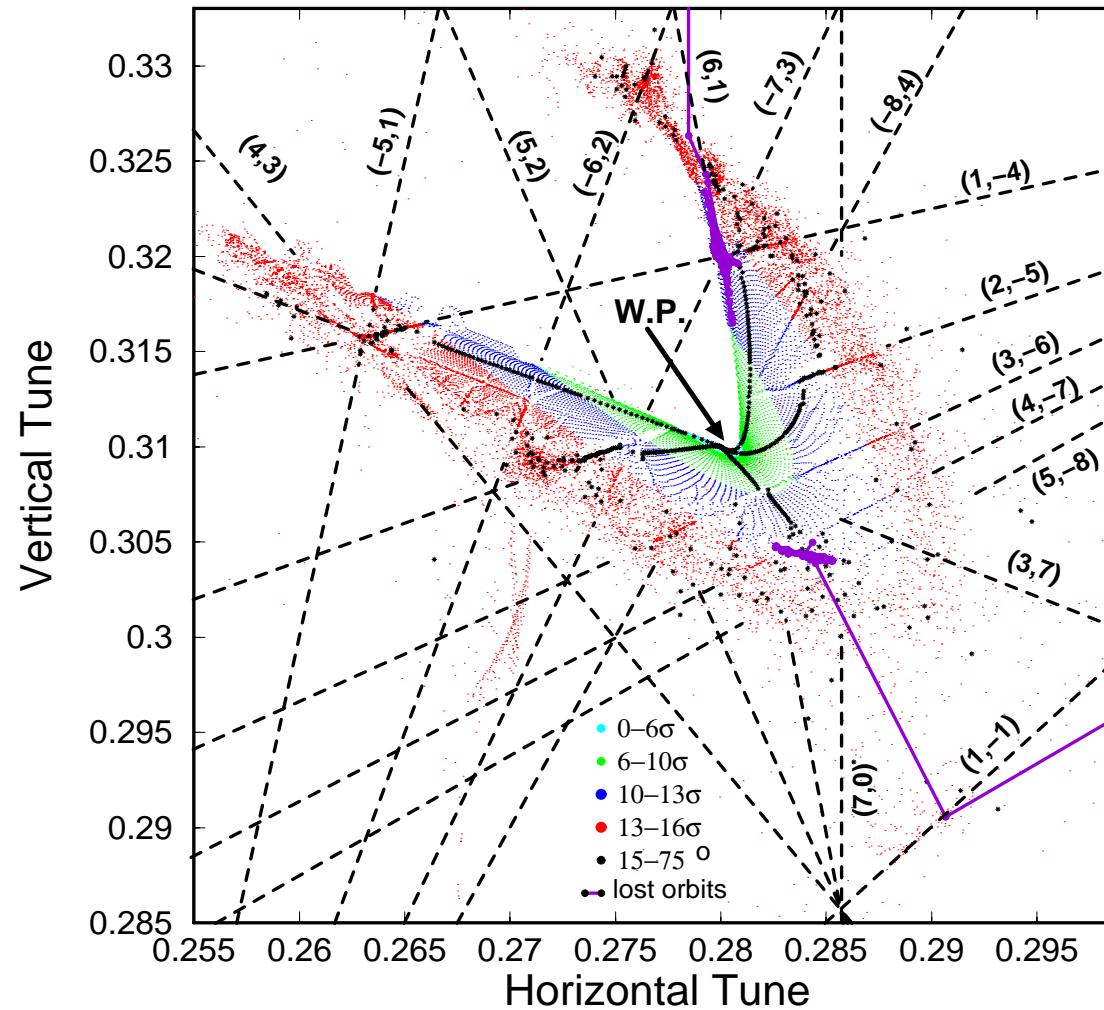
Version 5

- We experience a  $2\sigma$  loss in Dynamic Aperture

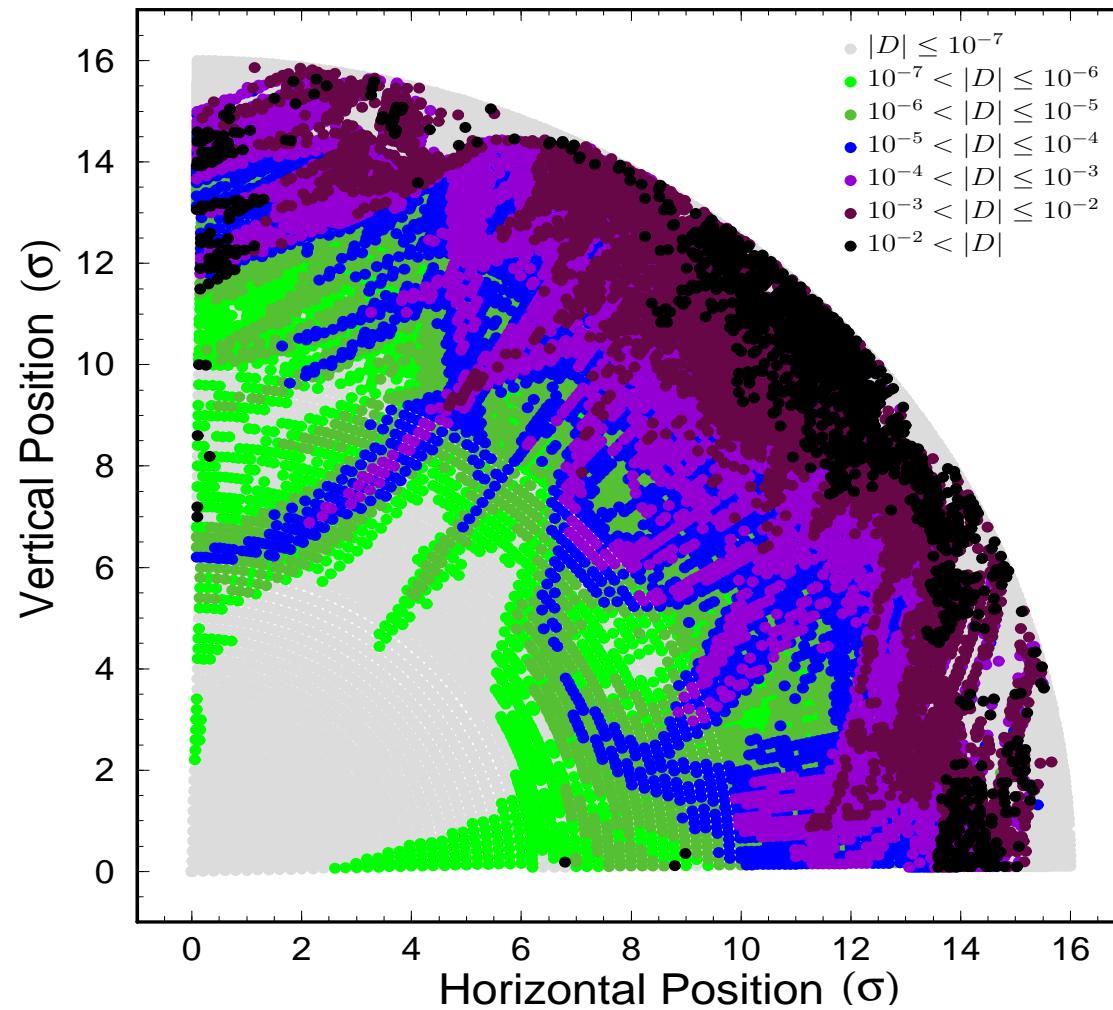


- Correction with “spool pieces”

## Effect of $a_4$



## Effect of $a_4$



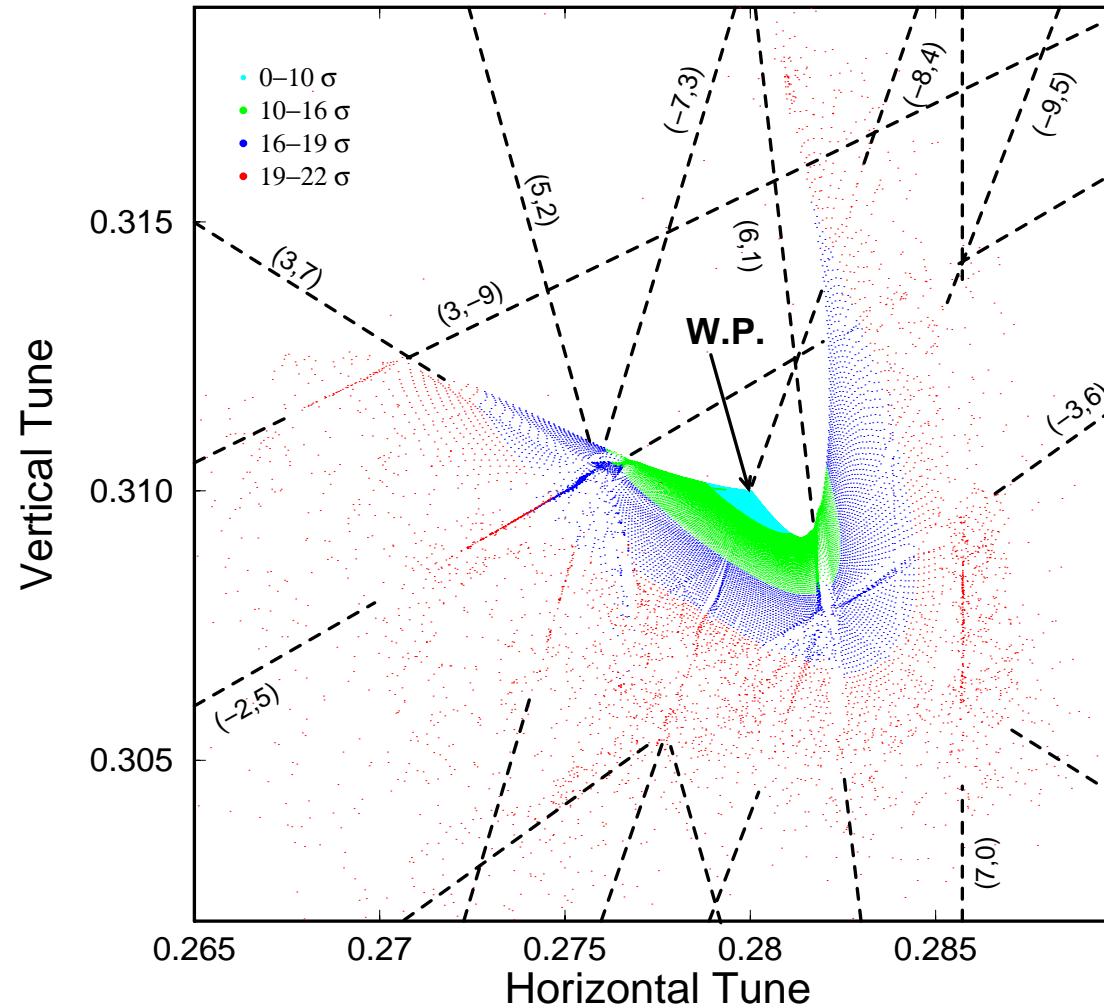


## Normal Octupole-Decapole Correction

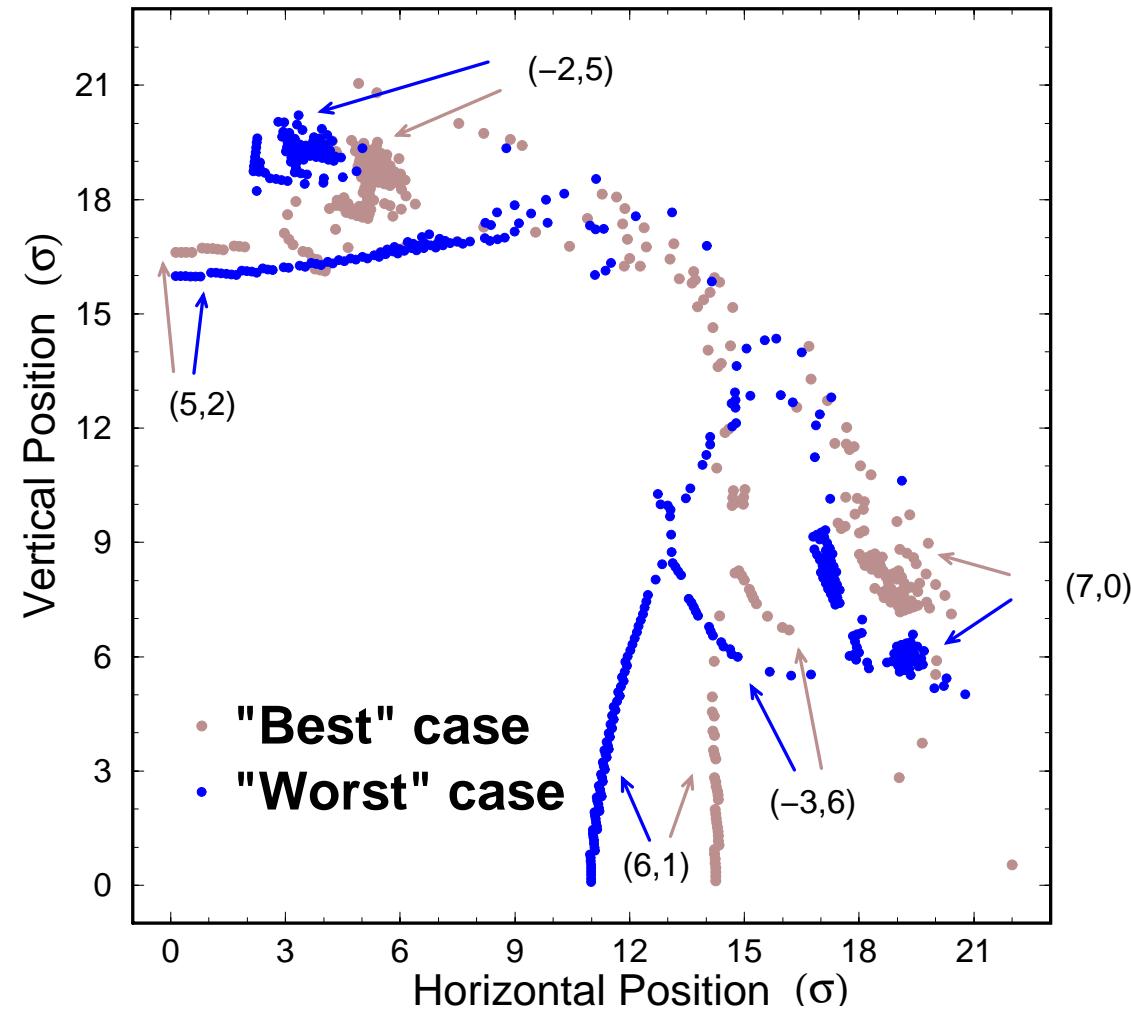
Five correction schemes for  $b_4-b_5$  with “spool pieces”

- s1.** Arc/Arc (reference case)
- s2.** Every 2nd dipole
- s3.** Every 2nd cell
- s4.** Every 2nd dipole and swap of the dipole types in every 2nd cell
- s5.** As s4 with dipole types inverted

## $b_4-b_5$ correction

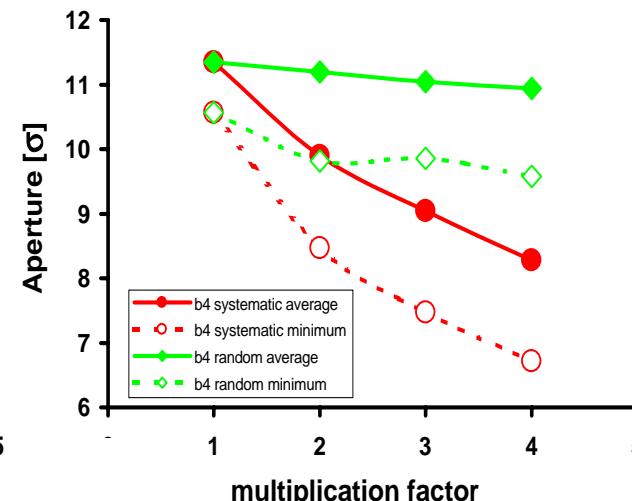
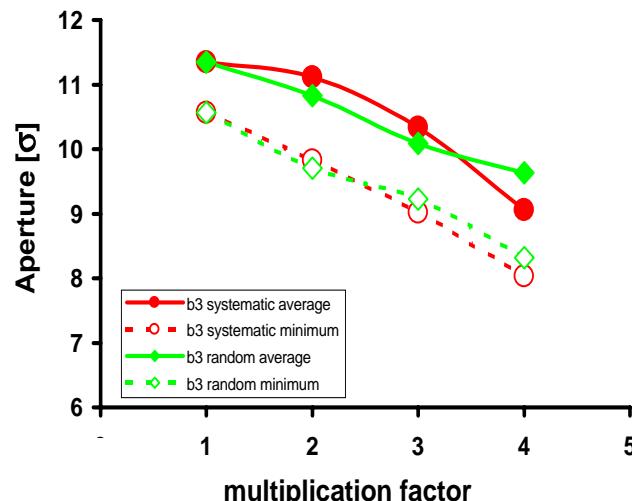


## $b_4$ - $b_5$ correction

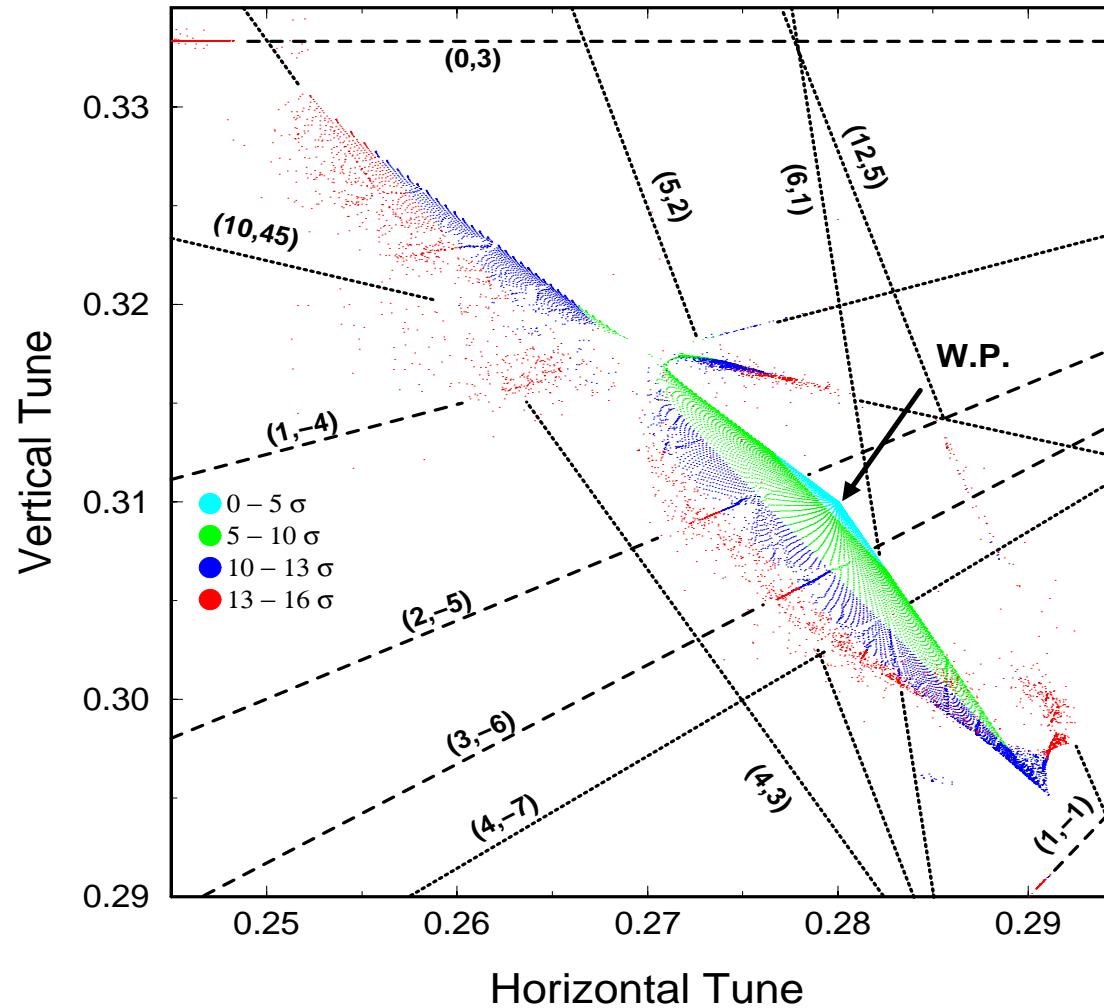


## Study of Decay and Snap-back

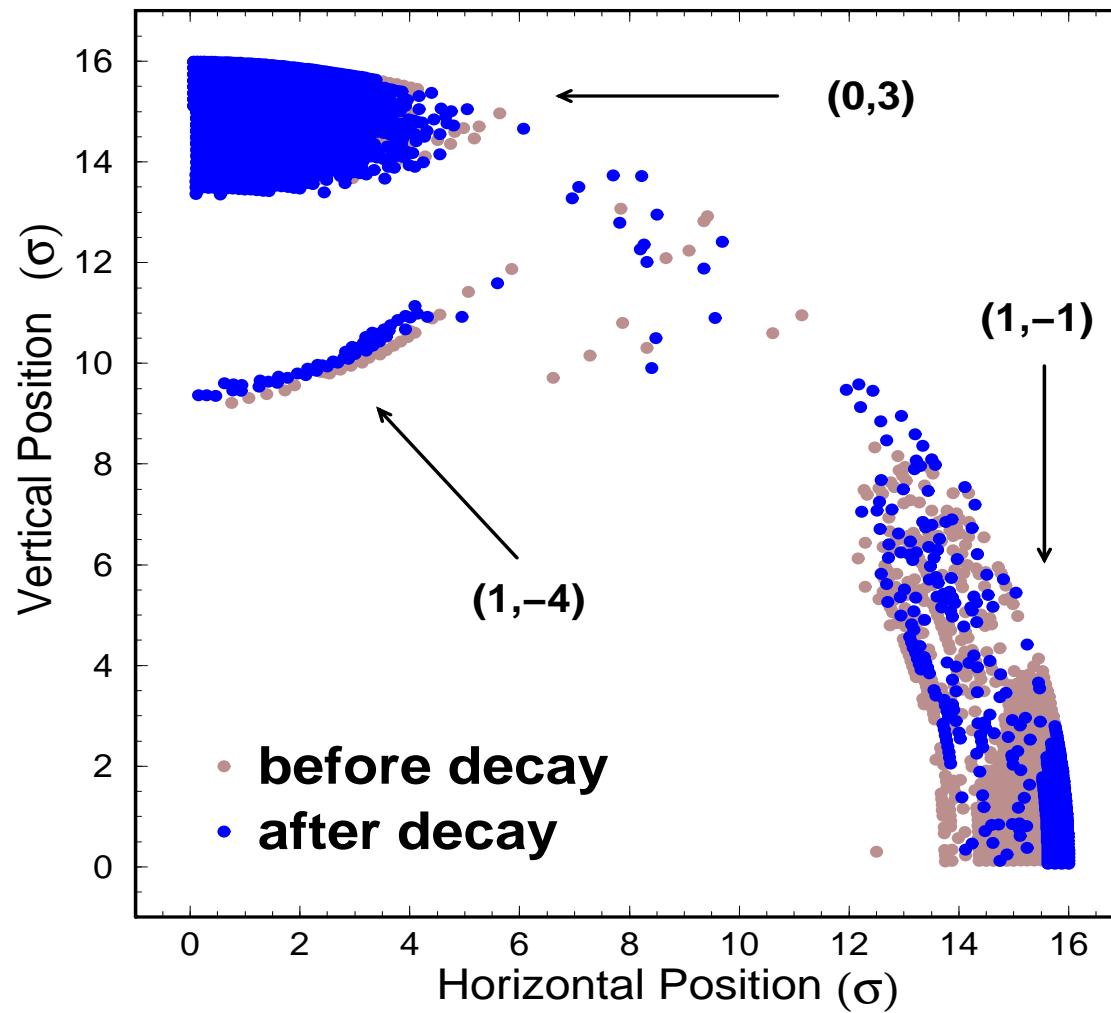
- Slow flux creep + current redistribution
- Small Dependence of Dynamic Aperture (LHC optics Version 6)



## Decay and Snap-back

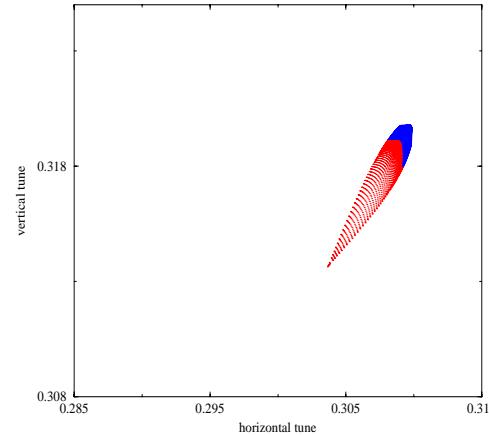


## Decay and Snap-back

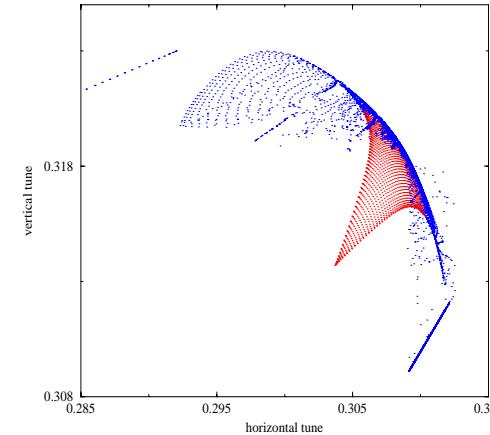


## Study of Beam-Beam Effect (with F. Zimmerman)

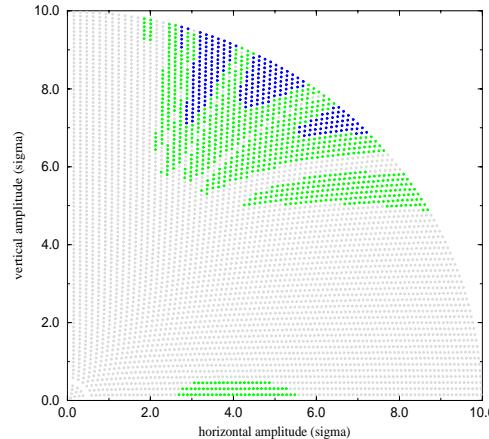
head-on collisions only



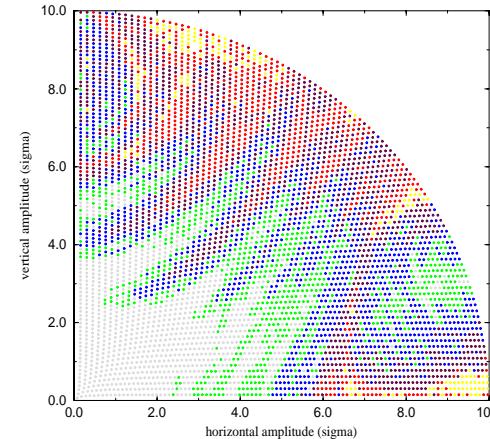
head-on plus long-range collisions



head-on collisions only



head-on plus long-range collisions





## Conclusion - Perspectives

- Global Dynamics in Frequency Maps
- Resonance Structure + Diffusion (Diffusion maps)
- Diffusion coefficient → Quality factor
- Guide before long term tracking
- Study of various LHC problems



## Perspectives

- Theoretical challenges:
  - a. Frequency maps for the 6D Phase space
  - b. Correlation between Diffusion Coefficient and Dynamic Aperture
  - c. Statistics for the Understanding Diffusion
  - d. Frequency Maps versus Normal forms
- Accelerator Dynamics Problems:
  - a. Choice of the Working Point
  - b. Small Variation of the Working Point
  - c. Study of Different Mechanisms Inducing Instabilities (ripple, beam-beam, etc.)